

## ANOTHER LOOK AT 8x8 MAGIC KNIGHT'S TOURS

In the last two issues, we have been looking at magic knight's tours within a 4x4x4 cube, and we have noted some tours found by Guenter Stertenbrink and Awani Kumar which are not merely magic along the principal space diagonals but "pandagonally magic" (magic along the broken space diagonals as well). In contrast, there is no tour on the 8x8 board which is even diagonally magic. However, it occurs to me that in looking for such a tour we may have been attempting something a little unnatural. The numbers from 1 to 64 add up to  $8 \times 260$ , so it is entirely reasonable to look for arrangements in which each of the eight rows and each of the eight columns adds to 260. But a knight's tour puts eight odd numbers on one long diagonal and eight even numbers on the other, and the odd numbers between 1 and 64 don't add to  $4 \times 260$ ; they add to  $4 \times 256$ , and the even numbers add to  $4 \times 264$ . Might it therefore be more realistic to look for tours which are "diagonally quasi-magic" (rows and columns add to 260, odd diagonal adds to 256, even diagonal to 264)?

As was reported in VC 43, a computer search in 2003 showed there to be 140 basic knight's tours which were row-and-column magic. Each of these basic tours can be placed on a chessboard in 16 different ways (it can be placed in any one of eight orientations, and can be numbered from either end), giving a grand total of 2240 arithmetically different tours, but a tabulation of the 140 basic tours is sufficient to give complete information. Furthermore, some closed loop tours share a common path, differing only in their starting points, and this reduces the number of tours that need be separately tabulated to 108. 96 of these were given by George Jelliss in *Chessics* 26 (plus one interloper which was exposed a few issues later), five more were published in an article by Tom Marlow in the May 1988 issue of *The Problemist* (two of them having appeared the previous year in issue 1 of *The Games and Puzzles Journal*), and the remaining seven are in VC 43. George posted a complete catalogue on his Knight's Tour Notes web site <[www.ktn.freeuk.com](http://www.ktn.freeuk.com)>, but not everybody has access to the Internet, and I at least much prefer reading a book or magazine to looking at a computer screen. It therefore seems to me that VC might usefully devote a couple of pages to reprinting this catalogue, now that it is definitive, and it appears on pages 52-53.

With all 140 tours in front of us, we can look for those which are "diagonally quasi-magic" as defined above. There turn out to be seven: tours 1, 4 (tour 1 starting from e6), 40, 41, 59, 60, and 62. Furthermore, tours 1, 4, and 62 have the additional property that each odd broken diagonal which is parallel to the main odd diagonal (c8-h3/a2-b1, e8-h5/a4-d1, g8-h7/a6-f1) adds to the same value 256, and each even broken diagonal which is parallel to the main even diagonal adds to 264. This elegant property seems a sufficient reason to extract these tours from the catalogue and to present them in full-size type :

15 30 1 50 27 36 63 38	27 30 51 40 53 2 15 42
2 51 14 29 64 39 26 35	50 39 28 31 14 41 54 3
31 16 49 4 33 28 37 62	29 26 37 52 1 56 43 16
52 3 32 13 40 61 34 25	38 49 32 25 44 13 4 55
17 42 53 48 5 24 11 60	23 36 45 12 57 64 17 6
54 45 20 41 12 57 8 23	48 11 24 33 20 5 58 61
43 18 47 56 21 6 59 10	35 22 9 46 63 60 7 18
46 55 44 19 58 9 22 7	10 47 34 21 8 19 62 59

Tours 1 and 4 (from e6)

Tour 62

Tour 62 is "rotationally symmetric" (diametrically opposite values always differ by 32), so starting this tour from d3 doesn't produce a different tour; it merely gives the same tour rotated through 180 degrees. And of course the summation properties are shared by the tours numbered in reverse from e7 (tour 1), c5 (tour 4), or f4 (tour 62), though the relevant odd diagonals now run NE-SW and the relevant even diagonals NW-SE.

These tours were discovered by C. F. Jaenisch. Tour 62 appeared in *The Chess Monthly* in 1859, and tours 1 and 4 in volume 2 of his *Traité des Applications de l'Analyse Mathématique au Jeu des Échecs*, St Petersburg 1862. However, while he stresses the fact that the long diagonals together add to 520, the more elegant property highlighted here seems to have escaped his notice. Nor does it seem to have been noticed by H. J. R. Murray, nor (by implication) by those whose work Murray used. I cannot believe it has genuinely remained unspotted until now, but at the moment I am not aware of any evidence to the contrary.

On a flat board, the tours above (rows and columns adding to 260, long odd diagonal and parallel odd broken diagonals adding to 256, long even diagonal and parallel even broken diagonals adding to 264) are the best we can do. The question however occurs to me: on a cylinder or torus, can we find a tour such that all the odd diagonals in both directions add to 256 and all the even diagonals in both directions to 264? Alternatively, can some mathematician prove the task to be impossible?

The catalogue that follows relies heavily on manuscript 106 of the H. J. R. Murray papers in the Bodleian Library and on a further unpublished Murray manuscript *The Magic Knight's Tours, a Mathematical Recreation* (Bodleian shelfmark MS Eng d.2370). I am grateful to the Bodleian for access to these and to other Murray papers, to Cambridge University Library for access to the Jaenisch book, and to George Jelliss for a photocopy made by Ken Whyld of the relevant pages from *The Chess Monthly*. Additionally, and to everybody's benefit, George has posted a chronological knight's tour bibliography on the Knight's Tour Notes web site, and he presented the historical material from Murray's *Magic Knight's Tours* manuscript mentioned above, with commentary, in issues 14 and 15 of *The Games and Puzzles Journal*.

Murray listed the tours in approximately chronological order, but in *Chessics* 26 George classified them by the separation of their end points, and I have followed him. The closed loop tours come first, then the tours with end points (0,1) apart, then (0,3) apart, and so on. The closed

loop tours are further divided into those admitting intermediate starting points (tours 1-47), those which are rotationally symmetric (tours 46-62, tours 46-47 being in both groups), and one other (tour 63). A rotationally symmetric tour can of course be renumbered starting from the square numbered 33, but this merely rotates it through 180 degrees. Similarly, the pattern of tours 46-47 only gives two distinct tours, those starting at e8 and d3; the tours starting at d1 and e6 are rotations of these.

As regards orientation and selection of a starting point, wherever possible I have chosen a representative in which the tour starts from the top edge of the board, and it always starts from a light square (so the diagonal a8-h1 is always odd). The tours within each group therefore tend to appear in an order slightly different from George's, and the table alongside links my numbers to his. Dates are normally those of publication (Beverley's tour was actually discovered in 1847, all those of Wenzelides in 1849, and at least the first two of Jaenisch's in 1858), but there may be minor errors during the period 1876-1885.

By classifying Murray's manuscript catalogue and publishing it in *Chessics* 26, George did much to encourage others to explore the subject and to fill in the gaps. The present catalogue, being the result of an exploration now complete, is unlikely to stimulate further work in the same way, but it may serve to commemorate

the closure of an exercise which took over a hundred and fifty years and occupied some fine brains. It seems appropriate that it should appear in a magazine which George himself founded.

1-5	00a	60	12k	87	07a	114	27a
6-9	00b	61	12o	88	14c	115	27b
10-13	00c	62	12n	89	14f	116	27d
14-17	00d	63	12p	90	14b	117	27c
18-21	00e	64	01g	91	14a	118	27e
22-25	00f	65	01b	92	14e	119	27f
26-29	00g	66	01c	93	14d	120	27g
30-33	00h	67	01d	94	16a	121	27h
34-35	00i	68	01e	95	23c	122	27t
36-37	00j	69	01f	96	23d	123	27q
38-39	00l	70	01a	97	23q	124	27s
40-41	00k	71	01h	98	23f	125	27p
42-43	00n	72	01i	99	23g	126	27r
44-45	00o	73	03c	100	23h	127	27i
46-47	00m	74	03d	101	23e	128	27j
48	12e	75	03b	102	23i	129	27k
49	12f	76	03a	103	23l	130	27l
50	12h	77	03f	104	23k	131	27m
51	12g	78	03e	105	23j	132	27n
52	12a	79	03g	106	23a	133	27o
53	12b	80	05g	107	23b	134	34e
54	12c	81	05a	108	23n	135	34a
55	12d	82	05b	109	23m	136	34d
56	12m	83	05f	110	23o	137	34c
57	12j	84	05d	111	23p	138	34b
58	12i	85	05c	112	25a	139	34g
59	12l	86	05e	113	25b	140	34f

Table linking my numbers to George's web catalogue numbers

**Sources in chronological order.** **114:** W. Beverley, *The London and Edinburgh Philosophical Magazine and Journal of Science*, August 1848 (Murray). **52:** C. Wenzelides, *Schachzeitung*, February-March 1849 (Murray). **115:** Ditto, July 1849 (Murray). **6-9:** Shri Krishna Udayar, King of Karnataka, on a silk carrying the date 31 July 1852 (Murray, also Akenhead, *The Fairy Chess Review*, April 1947). As with all transliterations of names from non-Latin scripts, the spelling in English accounts varies, and I am following S. R. Iyer, *Indian Chess*, Delhi 1982. Shri Krishna Udayar appears to be more usually described as the Rajah of Mysore, but again I am following Iyer. **46-47, 48, 53, 56:** Wenzelides, *Schachzeitung*, May 1858 (Murray). **61, 62:** C. F. Jaenisch, *The Chess Monthly*, April-June 1859. **1-5, 18-21, 116, 117:** Jaenisch, *Traité des Applications de l'Analyse Mathématique au Jeu des Échecs*, volume 2, St Petersburg 1862. Tour 2 (tour 1 starting from f1) appears to have been overlooked by Jaenisch, and according to Murray was given by Exner in 1876.

**81:** C. Bouvier, dated 1876 by Murray. **127, 134, 135:** Dr Exner, *Progr. des Gymn. zu Hirschberg*, 1876 (Murray). **82:** E. C. Caldwell, *English Mechanic and World of Science*, 1879 (Murray, confirmed by Jelliss). **34-35:** Published anonymously, dated 1880 by Murray. **54:** -. Béliqne, dated 1880 by Murray. **128:** Ditto, dated 1881. **10-13, 83, 129, 130:** E. Francony, dated 1881 by Murray. **57, 58, 84:** Bouvier, dated 1882 by Murray. **59, 60, 85, 86, 106, 131:** Francony, dated 1882 by Murray. **49...51, 55:** Paul de Hijo (pen name of Abbé Jolivald), dated 1882 by Murray. **90, 91:** Count Ligondès, dated 1882 by Murray. **38-39:** Bouvier, dated 1883 by Murray. **40-41:** Francony, dated 1883 by Murray. **22-25, 26-29, 65, 66, 70, 73, 74, 76...78, 80, 88, 95, 96, 98...105, 108, 109, 112, 136...140:** Ligondès, dated 1883 by Murray. **14-17, 36-37, 118, 123...126:** C. E. Reuss, dated 1883 by Murray. **93:** M. A. Feisthamel, dated 1884 by Murray. **107:** Ligondès, dated 1884 by Murray. **119:** M. Wihnyk, *Schachzeitung* 1885 (Murray).

Everything in the preceding paragraph is based on manuscript 106 of the Murray papers in the Bodleian Library, but there are minor inconsistencies between the dates given there and those implied by the summary in his *Magic Knight's Tours* manuscript, and it may be that one or two of them are in error by a year or so. The summary in the *Magic Knight's Tours* manuscript, although slightly imprecise (Murray was well over 80 when he wrote it), further implies that most if not all of the tours given without source first appeared in Feisthamel's column in *Le Siècle*, so an examination of this would clarify the matter, but I have not tried to make one. Murray notes that Jolivald's tours also appeared in his 1882 book *Le Problème du Cavalier des échecs d'après les méthodes qui donnent la symétrie par rapport au centre*, and those of Ligondès in a privately printed work he produced in 1884.

Every tour down to here was reported by General Parmentier in papers presented to the 1891 and 1892 meetings of the French Association pour l'Avancement des Sciences and subsequently published by him, tours 6-9 being attributed to Francony (Murray).

**68:** -. Grossetaitte, *Figaro*, 1896 (Murray). **30-33:** Ligondès, *La Mode du Petit Journal*, 1906 (Murray). **110:** Ditto, 1910 (Murray). **111:** Ditto, 1911 (Murray). Every tour down to here appeared in M. B. Lehmann's book *Neue Mathematische Spiele*, fourth edition 1932, tours 6-9 still being attributed to Francony (Murray).

**94:** M. B. Lehmann, *Le Sphinx*, August 1933 (Murray). **120:** H. J. R. Murray, *The Problemist Fairy Chess Supplement*, February 1936. **121:** Ditto, April 1936. **132, 133:** Murray, *The Fairy Chess Review*, August 1936. **63, 67, 69:** Ditto, November 1939. **79:** Ditto, June 1940. Every tour down to here appeared in *Chessics* 26.

**64, 97:** T. W. Marlow, *The Games and Puzzles Journal* 1, September-October 1987. **71, 75, 113:** Marlow, *The Problemist*, January 1988. **72, 92:** T. S. Roberts, *Variant Chess* 43, November 2003 (previously published on the Internet). **42-43, 44-45, 87, 89, 122:** H. Mackay, J.-C. Meyrignac, G. Stertenbrink, *Variant Chess* 43 (previously published on the Internet).

1-5 Jaenisch 1862	6-9 Udayar 1852	10-13 Francony 1881	14-17 Reuss 1883	18-21 Jaenisch 1862	22-25 Ligondès 1883
15 30 1 50 27 36 63 38 2 51 14 29 64 39 26 35 31 16 49 4 33 28 37 62 52 3 32 13 40 61 34 25 17 42 53 48 5 24 11 60 54 45 20 41 12 57 8 23 43 18 47 56 21 6 59 10 46 55 44 19 58 9 22 7	63 6 27 36 1 38 59 30 26 35 64 5 60 29 2 39 7 62 33 28 37 4 31 58 34 25 8 61 32 57 40 3 47 10 49 24 41 16 55 18 50 23 46 9 56 19 42 15 11 48 21 52 13 44 17 54 22 51 12 45 20 53 14 43	63 6 27 36 1 38 59 30 26 35 64 5 60 29 2 39 7 62 33 28 37 4 31 58 34 25 8 61 32 57 40 3 9 48 21 52 13 44 17 56 24 51 10 45 20 53 14 41 47 10 49 22 43 16 55 18 50 23 46 11 54 19 42 15	63 6 27 36 1 38 59 30 26 35 64 5 60 29 2 39 7 62 33 28 37 4 31 58 34 25 8 61 32 57 40 3 9 48 23 52 13 42 17 56 24 51 10 45 20 53 14 41 47 22 49 12 53 16 43 18 50 11 46 21 44 19 54 15	63 22 15 40 1 42 59 18 14 39 64 21 60 17 2 43 37 62 23 16 41 4 19 58 24 13 38 61 20 57 44 3 11 36 25 52 29 46 5 56 26 51 12 33 8 55 30 45 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31	63 22 15 40 1 42 59 18 14 39 64 21 60 17 2 43 23 62 37 16 41 4 19 58 38 13 24 61 20 57 44 3 25 36 11 52 29 46 5 56 12 51 26 33 8 55 30 45 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31
26-29 Ligondès 1883	30-33 Ligondès 1906	34-35 Anonymous 1880	36-37 Reuss 1883	38-39 Bouvier 1883	40-41 Francony 1883
63 22 15 40 1 42 59 18 14 39 64 21 60 17 2 43 37 62 23 16 41 58 19 4 24 13 38 61 20 3 44 57 51 36 25 12 45 56 5 30 26 11 52 33 8 29 46 55 35 50 9 28 53 48 31 6 10 27 34 49 32 7 54 47	43 50 1 24 5 48 63 26 2 23 44 49 64 25 6 47 51 42 21 4 45 8 27 62 22 3 52 41 28 61 46 7 39 54 19 20 9 14 35 60 30 19 40 53 36 59 10 13 55 38 17 32 57 12 15 34 18 31 56 37 16 33 58 11	63 6 55 16 1 42 59 18 54 15 64 5 60 17 2 43 7 62 13 56 41 4 19 58 14 53 8 61 20 57 44 3 51 12 25 36 45 40 21 30 26 35 52 9 24 29 46 39 11 50 33 28 37 48 31 22 34 27 10 49 32 23 38 47	63 6 27 36 1 38 59 30 26 35 64 5 60 29 2 39 7 62 33 28 37 4 31 58 34 25 8 61 32 57 40 3 9 48 11 52 13 54 17 56 24 51 10 45 20 53 14 41 47 10 49 12 53 16 55 18 50 23 46 21 44 19 42 15	23 46 61 18 7 44 59 2 62 19 24 45 60 1 6 43 47 22 63 8 17 42 3 58 20 25 36 41 64 5 16 53 37 48 21 32 9 52 57 4 26 35 10 49 40 31 54 15 11 38 33 28 13 56 51 30 34 27 12 39 50 29 14 55	23 46 43 18 7 62 59 2 44 19 24 63 42 1 6 61 47 22 45 8 17 60 3 58 20 25 36 41 64 5 16 53 37 48 21 32 9 52 57 4 26 35 28 49 40 13 54 15 29 38 33 10 31 56 51 12 34 27 30 39 50 11 14 55
42-43 MMS 2003	44-45 MMS 2003	46-47 Wenzelides 1858	48 Wenzelides 1858	49 Jolivald 1882	50 Jolivald 1882
63 2 59 24 57 14 19 22 60 25 62 1 20 23 56 13 3 64 27 58 15 54 21 18 26 61 4 53 32 17 12 55 39 28 33 16 37 52 45 10 34 5 38 31 42 11 48 51 29 40 7 36 49 46 9 44 6 35 30 41 8 43 50 47	31 2 27 56 25 14 51 54 28 57 30 1 52 55 24 13 3 32 59 26 15 22 53 50 58 29 4 21 64 49 12 23 39 60 33 16 37 20 45 10 34 5 38 63 42 11 48 19 61 40 7 36 17 46 9 44 6 35 62 41 8 43 18 47	47 6 63 24 1 50 43 26 62 23 48 5 44 25 2 51 21 46 7 64 49 4 27 42 8 61 22 45 28 41 52 3 35 20 9 60 13 54 29 40 10 59 36 17 32 39 14 53 19 34 57 12 37 16 55 30 58 11 18 33 56 31 38 15	43 26 1 48 5 24 63 50 2 47 44 25 64 49 6 23 45 42 27 4 21 8 51 62 28 3 46 41 52 61 22 7 39 54 29 20 9 14 35 60 30 19 40 53 36 59 10 13 55 38 17 32 57 12 15 34 18 31 56 37 16 33 58 11	43 46 1 24 41 20 63 22 2 25 42 45 64 23 40 19 47 44 27 4 17 38 21 62 26 3 48 37 28 61 18 39 7 50 29 60 5 16 35 58 30 53 6 49 36 59 12 15 51 8 55 32 13 10 57 34 54 31 52 9 56 33 14 11	43 46 1 26 41 18 63 22 2 27 42 45 64 23 40 17 47 44 25 4 19 38 21 62 28 3 52 37 24 61 16 39 7 48 29 56 5 20 35 60 30 53 6 51 36 57 12 15 49 8 55 32 13 10 59 34 54 31 50 9 58 33 14 11
51 Jolivald 1882	52 Wenzelides 1849	53 Wenzelides 1858	54 Bèlignè 1880	55 Jolivald 1882	56 Wenzelides 1858
43 18 1 56 5 16 63 58 2 55 44 17 64 57 6 15 45 42 19 4 21 8 59 62 28 3 54 41 52 61 14 7 39 46 29 20 9 22 35 60 30 27 40 53 36 51 10 13 47 38 25 32 49 12 23 34 26 31 48 37 24 33 50 11	63 6 55 16 1 42 59 18 54 15 64 5 60 17 2 43 7 62 13 56 41 4 19 58 14 53 8 61 20 57 44 3 35 12 25 52 29 40 21 46 26 51 36 9 24 45 30 39 11 34 49 28 37 32 47 22 50 27 10 33 48 23 38 31	63 6 23 48 1 10 59 50 22 47 64 5 60 49 2 11 7 62 45 24 9 4 51 58 46 21 8 61 52 57 12 3 35 44 25 20 29 40 53 14 26 19 36 41 56 13 30 39 43 34 17 28 37 32 15 54 18 27 42 33 16 55 38 31	63 24 39 20 1 26 41 46 38 19 64 25 40 45 2 27 23 62 17 44 21 4 47 42 18 37 22 61 48 43 28 3 35 60 1 16 29 54 5 50 10 15 36 53 12 49 30 55 59 34 13 8 57 32 51 6 14 9 58 33 52 7 56 31	63 24 39 18 1 26 43 46 38 17 64 25 42 45 2 27 23 62 19 40 21 4 47 44 16 37 22 61 52 41 28 3 35 60 9 20 29 54 5 48 12 15 36 53 8 51 30 55 59 34 13 10 57 32 49 6 14 11 58 33 50 7 56 31	59 6 63 16 1 42 55 18 62 15 60 5 56 17 2 43 7 58 13 64 41 4 19 54 14 61 8 57 20 53 44 3 35 12 21 52 25 40 29 46 22 51 36 9 32 45 26 39 11 34 49 24 37 28 47 30 50 23 10 33 48 31 38 27
57 Bouvier 1882	58 Bouvier 1882	59 Francony 1882	60 Francony 1882	61 Jaenisch 1859	62 Jaenisch 1859
23 46 61 18 7 44 59 2 62 19 24 45 60 1 6 43 47 22 63 8 17 42 3 58 20 25 36 41 64 53 16 5 37 48 21 32 9 4 57 52 26 35 10 49 40 31 54 15 11 38 33 28 13 56 51 30 34 27 12 39 50 29 14 55	23 46 61 18 7 44 59 2 62 19 24 45 60 1 6 43 47 22 63 8 17 42 3 58 36 25 40 41 64 5 16 53 21 48 37 32 9 52 57 4 26 35 10 49 40 31 54 15 11 38 33 28 13 56 51 30 34 27 12 39 50 29 14 55	23 46 43 18 7 62 59 2 44 19 24 63 42 1 6 61 47 22 45 8 17 60 3 58 20 25 36 41 64 53 16 5 37 48 21 32 9 4 57 52 26 35 28 49 40 13 54 15 29 38 33 10 31 56 51 12 34 27 30 39 50 11 14 55	23 46 43 18 7 62 59 2 44 19 24 63 42 1 6 61 47 22 45 8 17 60 3 58 36 25 40 41 64 5 16 53 21 48 37 32 9 52 57 4 26 35 28 49 40 13 54 15 29 38 33 10 31 56 51 12 34 27 30 39 50 11 14 55	15 54 41 2 51 28 39 30 42 3 14 53 40 31 50 27 55 16 1 44 25 52 29 38 4 43 56 13 32 37 26 49 17 58 5 64 45 24 11 36 6 61 20 57 12 33 48 23 59 18 63 8 21 46 35 10 62 7 60 19 34 9 22 47	27 30 51 40 53 2 15 42 50 39 28 31 14 41 54 3 29 26 37 52 1 56 43 16 38 49 32 25 44 13 4 55 23 36 45 12 57 64 17 6 48 11 24 33 20 5 58 61 35 22 9 46 63 60 7 18 10 47 34 21 8 19 62 59
63 Murray 1939	64 Marlow 1987	65 Ligondès 1883	66 Ligondès 1883	67 Murray 1939	68 Grossetaite 1896
63 42 23 32 1 34 27 38 22 31 64 41 28 37 2 35 43 62 29 24 33 4 39 26 30 21 44 61 40 25 36 3 11 60 13 20 45 52 5 54 14 17 10 57 8 55 48 51 59 12 19 16 49 46 53 6 18 15 58 9 56 7 50 47	59 14 1 64 11 26 55 30 2 63 60 13 56 29 10 27 61 58 15 4 25 12 31 54 16 3 62 57 32 53 28 9 39 42 17 24 5 48 33 52 18 21 38 41 36 51 8 47 43 40 23 20 45 6 49 34 22 19 44 37 50 35 46 7	39 22 15 64 1 42 59 18 14 63 40 21 60 17 2 43 23 38 61 16 41 4 19 58 62 13 24 37 20 57 44 3 25 36 11 52 29 46 5 56 12 51 26 33 8 55 30 45 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31	39 22 15 64 1 42 59 18 14 63 40 21 60 17 2 43 23 38 61 16 41 58 19 4 62 13 24 37 20 3 44 57 25 36 11 52 29 46 5 46 12 51 26 33 8 45 30 55 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31	31 42 23 64 1 34 27 38 22 63 32 41 28 37 2 35 43 30 61 24 33 4 39 26 62 21 44 29 40 25 36 3 11 60 13 20 45 52 5 54 14 17 10 57 8 55 48 51 59 12 19 16 49 46 53 6 18 15 58 9 56 7 50 47	63 34 55 14 43 2 23 26 54 13 64 1 56 25 44 3 33 62 35 42 15 22 27 24 12 53 32 21 36 57 4 45 61 20 41 52 5 16 37 28 8 11 6 31 58 51 46 49 19 60 9 40 17 48 29 38 10 7 18 59 30 39 50 47
69 Murray 1939	70 Ligondès 1883	71 Marlow 1988	72 Roberts 2003	73 Ligondès 1883	74 Ligondès 1883
63 2 51 22 53 16 35 18 50 23 64 1 34 19 14 55 3 62 21 52 15 54 17 36 24 49 4 61 20 33 56 13 5 44 25 48 9 60 37 32 26 47 8 41 30 39 12 57 43 6 45 28 59 10 31 38 46 27 42 7 40 29 58 11	59 6 27 36 61 38 31 2 26 35 60 5 32 1 62 39 7 58 33 28 37 64 3 30 34 25 8 57 4 29 40 63 9 56 23 48 13 52 17 42 24 47 10 53 20 41 14 51 55 22 45 12 49 16 43 18 46 11 54 21 44 19 50 15	59 62 31 4 33 46 19 6 30 3 60 63 18 5 34 47 61 58 1 32 45 36 7 20 2 29 64 57 8 17 48 35 27 56 9 16 37 44 21 50 12 15 28 53 24 49 38 41 55 26 13 10 43 40 51 22 14 11 54 25 52 23 42 39	39 58 25 6 55 50 23 4 26 7 38 57 24 5 54 49 59 40 1 28 51 56 3 22 8 27 64 37 2 21 48 53 41 60 9 14 29 52 35 20 10 15 44 63 36 13 32 47 61 42 17 12 45 30 19 34 16 11 62 43 18 33 46 31	35 6 27 60 1 38 31 62 26 59 36 5 32 61 2 39 7 34 57 28 37 4 63 30 58 25 8 33 64 29 40 3 9 56 23 48 13 52 17 42 24 47 10 53 20 41 14 51 55 22 45 12 49 16 43 18 46 11 54 21 44 19 50 15	35 6 27 60 1 38 31 62 26 59 36 5 32 61 2 39 57 34 7 28 37 4 63 30 8 25 58 33 64 29 40 3 23 56 9 48 13 52 17 42 10 47 24 53 20 41 14 51 55 22 45 12 49 16 43 18 46 11 54 21 44 19 50 15
75 Marlow 1988	76 Ligondès 1883	77 Ligondès 1883	78 Ligondès 1883	79 Murray 1940	80 Ligondès 1883
31 6 35 60 1 10 55 62 34 59 32 5 56 61 2 11 7 30 57 36 9 4 63 54 58 33 8 29 64 53 12 3 19 22 37 52 13 28 43 46 38 51 18 21 44 47 14 27 23 20 49 40 25 16 45 42 50 39 24 17 48 41 26 15	43 2 47 24 49 26 39 30 46 23 44 1 40 29 50 27 3 42 21 48 25 52 31 38 22 45 4 41 32 37 28 51 5 20 57 64 9 16 53 36 60 63 8 17 56 33 10 13 19 6 61 58 15 12 35 54 62 59 18 7 34 55 14 11	63 22 15 40 17 42 59 2 14 39 64 21 60 1 18 43 23 62 37 16 41 20 3 58 38 13 24 61 4 57 44 19 25 36 11 52 29 46 5 56 12 51 26 33 8 55 30 45 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31	63 22 15 40 17 42 59 2 14 39 64 21 60 1 18 43 37 62 23 16 41 20 3 58 24 13 38 61 4 57 44 19 11 36 25 52 29 46 5 56 26 51 12 33 8 55 30 45 35 10 49 28 53 32 47 6 50 27 34 9 48 7 54 31	63 14 39 20 41 22 59 2 38 17 64 15 60 1 42 23 13 62 19 40 21 58 3 44 18 37 16 61 4 43 24 57 31 12 33 52 25 56 45 6 36 51 30 9 48 5 26 55 11 32 49 34 53 28 7 46 50 35 10 29 8 47 54 27	43 30 53 4 45 28 51 6 54 1 44 29 52 5 48 27 31 42 3 56 25 46 7 50 2 55 32 41 8 49 26 47 63 34 9 24 57 40 15 18 10 23 62 33 16 19 58 39 35 64 21 12 37 60 17 14 22 11 36 61 20 13 38 59
81 Bouvier 1876	82 Caldwell 1879	83 Francony 1881	84 Bouvier 1882	85 Francony 1882	86 Francony 1882
31 2 39 58 41 16 55 18 38 59 30 1 56 19 42 15 3 32 57 40 43 14 17 54 60 37 4 29 20 53 44 13 33 28 61 8 45 12 21 52 62 5 36 25 22 51 48 11 27 34 7 64 9 46 23 50 6 63 26 35 24 49 10 47	31 2 39 58 41 16 55 18 38 59 30 1 56 19 42 15 3 32 57 40 13 44 17 54 60 37 4 29 20 53 14 43 33 28 61 8 45 12 51 22 62 5 36 25 52 21 48 11 27 34 7 64 9 46 23 50 6 63 26 35 24 49 10 47	27 2 43 52 29 6 47 54 42 51 28 1 48 53 30 7 3 26 49 44 5 32 55 46 50 41 4 25 56 45 8 31 39 24 61 16 33 20 57 10 62 15 40 21 60 9 34 19 23 38 13 64 17 36 11 58 14 63 22 37 12 59 18 35	31 2 57 40 15 18 55 42 38 59 30 1 56 41 16 19 3 32 59 58 17 14 43 54 60 37 4 29 44 53 20 13 33 28 61 8 21 12 45 52 62 5 36 25 48 51 22 11 27 34 7 64 9 24 49 46 6 63 26 35 50 47 10 23	31 2 39 58 15 18 55 42 38 59 30 1 56 41 16 19 3 32 57 40 17 14 43 54 60 37 4 29 44 53 20 13 33 28 61 8 21 12 45 52 62 5 36 25 48 51 22 11 27 34 7 64 9 24 49 46 6 63 26 35 50 47 10 23	31 2 39 58 41 16 55 18 38 59 30 1 56 19 14 43 3 32 57 40 15 42 17 54 60 37 4 29 20 53 44 13 33 28 61 8 45 12 21 52 62 5 36 25 50 23 48 11 27 34 7 64 9 46 51 22 6 63 26 35 24 49 10 47

87 MMS 2003 51 54 1 6 49 22 43 34 2 5 50 53 44 35 48 23 55 52 7 4 21 46 33 42 8 3 56 45 36 41 24 47 57 62 9 20 25 32 37 18 10 13 58 61 40 19 28 31 63 60 15 12 29 26 17 38 14 11 64 59 16 39 30 27	88 Ligondès 1883 31 58 1 40 55 42 15 18 38 3 30 57 16 19 54 43 59 32 5 2 41 56 17 14 4 37 60 29 20 13 44 53 33 28 5 64 45 52 21 12 6 61 36 25 10 23 48 51 27 34 63 8 49 46 11 22 62 7 26 35 24 9 50 47	89 MMS 2003 27 54 1 48 29 52 15 34 2 47 28 53 16 33 30 51 55 26 45 4 49 14 35 32 46 3 56 17 44 31 50 13 7 42 25 64 5 36 19 62 24 57 6 43 18 63 12 37 41 8 59 22 39 10 61 20 58 23 40 9 60 21 38 11	90 Ligondès 1882 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 4 45 26 50 13 36 29 48 25 56 3 11 38 17 64 5 44 23 58 18 63 12 37 24 57 6 43 39 10 61 20 41 8 59 22 62 19 40 9 60 21 42 7	91 Ligondès 1882 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 4 45 26 50 13 36 29 48 25 56 3 11 38 17 64 5 44 23 58 18 63 12 37 24 57 6 43 39 10 61 20 41 8 59 22 62 19 40 9 60 21 42 7	92 Roberts 2003 27 24 31 52 1 54 37 34 30 51 28 25 36 33 2 55 23 26 49 32 53 4 35 38 50 29 22 13 48 39 56 3 11 14 41 64 5 20 47 58 42 63 12 21 40 57 6 19 15 10 61 44 17 8 59 46 62 43 16 9 60 45 18 7
93 Feisthamel 1884 47 22 39 28 1 50 43 30 38 25 48 23 44 29 2 51 21 46 27 40 49 4 31 42 26 37 24 45 32 41 52 3 15 20 33 60 5 64 9 54 36 59 14 17 12 53 6 63 19 16 57 34 61 8 55 10 58 35 18 13 56 11 62 7	94 Lehmann 1933 31 54 1 6 43 48 27 50 4 7 30 53 28 51 42 45 55 32 5 2 47 44 49 26 8 3 60 29 52 21 46 41 61 56 33 12 37 16 25 20 34 9 62 59 22 19 40 15 57 64 11 36 13 38 17 24 10 35 58 63 18 23 14 39	95 Ligondès 1883 43 50 1 26 63 48 23 6 2 27 44 49 24 5 62 47 51 42 25 4 45 64 7 22 28 3 52 41 8 21 46 61 53 40 13 32 57 36 9 20 14 29 56 37 12 17 60 35 39 54 31 16 33 58 19 10 30 15 38 55 18 11 34 59	96 Ligondès 1883 59 42 1 20 61 40 15 22 2 19 60 41 16 21 62 39 43 58 17 4 37 64 23 14 18 3 44 57 24 13 38 63 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11	97 Marlow 1987 43 48 1 50 63 18 31 6 2 51 44 47 30 5 62 19 45 42 49 4 17 64 7 32 52 3 46 41 8 29 20 61 39 10 53 16 57 24 33 28 54 13 40 9 36 27 60 21 11 38 15 56 23 58 25 34 14 55 12 37 26 35 22 59	98 Ligondès 1883 19 42 1 60 21 40 15 62 2 59 20 41 16 61 22 39 43 18 57 4 37 24 63 24 58 3 44 17 64 13 38 23 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11
99 Ligondès 1883 19 42 1 60 21 40 15 62 2 59 20 41 16 61 22 39 43 18 57 4 37 24 63 24 58 3 44 17 64 13 38 23 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11	100 Ligondès 1883 19 42 1 60 21 40 15 62 2 59 20 41 16 61 22 39 57 18 43 4 37 14 63 24 44 3 58 17 64 23 38 13 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11	101 Ligondès 1883 19 42 1 60 21 40 15 62 2 59 20 41 16 61 22 39 57 18 43 4 37 24 63 14 44 3 58 17 64 13 38 23 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11	102 Ligondès 1883 23 62 15 40 1 42 19 58 14 39 22 61 20 59 2 43 63 24 37 16 41 4 57 18 38 13 64 21 60 17 44 3 11 50 25 36 5 46 31 56 26 35 12 49 32 55 6 45 51 10 33 28 53 8 47 30 34 27 52 9 48 29 54 7	103 Ligondès 1883 23 62 15 40 1 42 19 58 14 39 22 61 20 59 2 43 63 24 37 16 41 4 57 18 38 13 64 21 60 17 44 3 11 50 25 36 5 46 31 56 26 35 12 49 32 55 6 45 51 10 33 28 53 8 47 30 34 27 52 9 48 29 54 7	104 Ligondès 1883 23 62 15 40 1 42 19 58 14 39 22 61 20 59 2 43 63 24 37 16 41 4 57 18 38 13 64 21 60 17 44 3 25 50 11 36 5 46 31 56 12 35 26 49 32 55 6 45 51 10 33 28 53 8 47 30 34 27 52 9 48 29 54 7
105 Ligondès 1883 23 62 15 40 1 42 19 58 14 39 22 61 20 59 2 43 63 24 37 16 41 4 57 18 38 13 64 21 60 17 44 3 25 50 11 36 5 46 31 56 12 35 26 49 32 45 6 55 51 10 33 28 53 8 47 30 34 27 52 9 48 29 54 7	106 Francony 1882 31 6 35 60 1 62 27 38 34 59 32 5 28 37 2 63 7 30 57 36 61 4 39 26 58 33 8 29 40 25 64 3 15 42 17 56 9 48 23 50 18 55 14 41 24 51 10 47 43 16 53 20 45 12 49 22 54 19 44 13 52 21 46 11	107 Ligondès 1884 59 6 27 36 1 62 31 38 26 35 60 5 32 37 2 63 7 58 33 28 61 4 39 30 34 25 8 57 40 29 64 3 9 56 23 48 13 52 17 42 24 47 10 53 20 41 14 51 55 22 45 12 49 16 43 18 46 11 54 21 44 19 50 15	108 Ligondès 1883 47 62 23 6 43 50 27 2 22 7 48 63 26 1 42 51 61 46 5 24 49 44 3 28 8 21 64 45 4 25 52 41 19 60 9 36 13 40 29 54 10 35 20 57 32 53 14 39 59 18 33 12 37 16 55 30 34 11 58 17 56 31 38 15	109 Ligondès 1883 47 62 23 6 43 50 27 2 22 7 48 63 26 1 42 51 61 46 5 24 49 44 3 28 8 21 64 45 4 25 52 41 35 60 9 20 53 40 29 14 10 19 36 57 32 13 54 39 59 34 17 12 37 56 15 30 18 11 58 33 16 31 38 55	110 Ligondès 1910 39 62 23 18 47 42 27 2 22 17 40 63 26 1 48 43 61 38 19 24 41 46 3 28 16 21 64 33 8 25 44 49 37 60 9 20 45 56 29 4 12 15 34 57 32 7 50 53 59 36 13 10 55 52 5 30 14 11 58 35 6 31 54 51
111 Ligondès 1911 43 28 47 4 53 30 49 6 46 3 44 29 48 5 54 31 27 42 1 52 25 56 7 50 2 45 26 41 8 51 32 55 39 20 63 24 33 14 57 10 62 23 40 13 64 9 34 15 19 38 21 60 17 36 11 58 22 61 18 37 12 59 16 35	112 Ligondès 1883 59 42 1 20 61 40 15 22 2 19 60 41 16 21 62 39 43 58 17 4 37 14 63 24 18 3 44 57 24 63 38 13 31 56 5 48 9 36 25 50 6 45 32 53 28 49 12 35 55 30 47 8 33 10 51 26 46 7 54 29 52 27 34 11	113 Marlow 1988 11 54 15 44 1 46 39 50 14 43 12 53 40 49 2 47 55 10 41 16 45 38 51 4 42 13 56 9 52 3 48 37 63 28 17 32 57 36 5 22 18 31 64 25 8 21 58 35 27 62 29 20 33 60 23 6 30 19 26 61 24 7 34 59	114 Beverley 1848 1 30 47 52 5 28 43 54 48 51 2 29 44 53 6 27 31 46 49 4 25 8 55 42 50 3 32 45 56 41 26 7 33 62 15 20 9 24 39 58 16 19 34 61 40 57 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11	115 Wenzelides 1849 1 30 47 52 5 28 43 54 48 51 2 29 6 27 42 55 31 46 49 4 53 44 25 8 50 3 32 45 56 41 26 7 33 62 15 20 37 60 9 24 16 19 34 61 12 21 40 57 63 14 17 36 59 38 23 10 18 35 64 13 22 11 58 39	116 Jaenisch 1862 1 30 47 52 5 28 43 54 48 51 2 29 44 53 6 27 31 46 49 4 27 6 55 42 50 3 32 45 56 41 8 25 33 62 15 20 9 24 57 40 16 19 34 61 38 59 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11
117 Jaenisch 1862 1 30 47 52 27 54 43 6 48 51 2 29 44 5 26 55 31 46 49 4 53 28 7 42 50 3 32 45 8 41 56 25 33 62 15 20 57 24 9 40 16 19 34 61 12 37 58 23 63 14 17 36 21 60 39 10 18 35 64 13 38 11 22 59	118 Reuss 1883 1 30 47 52 5 28 43 54 48 51 2 29 44 53 6 27 31 46 49 4 7 26 55 42 50 3 32 45 40 57 24 9 33 62 15 20 25 8 41 56 16 19 34 61 58 39 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11	119 Wihnyk 1885 1 30 47 52 5 28 43 54 48 51 2 29 44 53 6 27 31 46 49 4 7 26 55 42 50 3 32 45 56 41 8 25 33 62 15 20 9 24 57 40 16 19 34 61 58 39 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11	120 Murray 1936 1 30 47 52 3 28 45 54 48 51 2 29 46 53 4 27 31 8 49 42 25 6 55 44 50 41 32 7 56 43 26 5 9 62 39 20 33 24 15 58 40 19 61 16 57 34 23 63 38 17 12 21 36 59 14 18 11 64 37 60 13 22 35	121 Murray 1936 1 30 47 52 3 28 45 54 48 51 2 29 46 53 4 27 31 8 49 42 25 6 55 44 50 41 32 7 56 43 26 5 39 62 9 20 33 24 15 58 10 19 40 61 16 57 34 23 63 38 17 12 21 36 59 14 18 11 64 37 60 13 22 35	122 MMS 2003 1 30 47 52 3 28 45 54 48 51 2 29 46 53 4 27 31 12 49 14 21 34 55 44 50 37 32 35 60 15 26 5 11 62 13 20 33 22 43 56 38 19 36 61 16 59 6 25 63 10 17 40 23 8 57 42 18 39 64 9 58 41 24 7
123 Reuss 1883 47 54 1 28 5 52 43 30 2 27 48 53 44 29 6 51 25 46 55 4 49 8 31 42 56 3 26 45 32 41 50 7 15 24 33 60 37 20 9 62 34 57 16 21 12 61 40 19 23 14 59 36 17 38 63 10 58 35 22 13 64 11 18 39	124 Reuss 1883 47 54 1 28 5 52 43 30 2 27 48 53 44 29 6 51 25 46 55 4 49 42 31 8 56 3 26 45 32 7 50 41 15 24 33 60 37 20 9 62 34 57 16 21 12 61 40 19 23 14 59 36 17 38 63 10 58 35 22 13 64 11 18 39	125 Reuss 1883 47 54 1 28 5 52 43 30 2 27 48 53 44 29 6 51 55 46 25 4 49 8 31 42 26 3 56 45 32 41 50 7 15 24 33 60 37 20 9 62 34 57 16 21 12 61 40 19 23 14 59 36 17 38 63 10 58 35 22 13 64 11 18 39	126 Reuss 1883 47 54 1 28 5 52 43 30 2 27 48 53 44 29 6 51 55 46 25 4 49 42 31 8 26 3 56 45 32 7 50 41 15 24 33 60 37 20 9 62 34 57 16 21 12 61 40 19 23 14 59 36 17 38 63 10 58 35 22 13 64 11 18 39	127 Exner 1876 43 48 27 52 1 54 5 30 26 51 44 47 6 29 2 55 45 42 49 28 53 4 31 8 50 25 46 41 32 7 56 3 15 40 19 24 57 62 9 34 20 23 16 37 12 33 58 61 39 14 21 18 63 60 35 10 22 17 38 13 36 11 64 59	128 Bèligné 1881 7 54 43 28 1 30 47 50 42 27 6 53 48 51 2 31 25 8 55 44 29 4 49 46 56 41 26 5 52 45 32 3 9 24 39 60 33 62 15 18 40 57 10 21 16 19 34 63 23 38 59 12 61 36 17 14 58 11 22 37 20 13 64 35
129 Francony 1881 7 54 43 28 1 30 47 50 42 27 6 53 48 51 2 31 25 8 55 44 29 4 49 46 56 41 26 5 52 45 32 3 9 24 39 60 33 62 15 18 40 57 10 21 16 19 34 63 23 38 59 12 61 36 17 14 58 11 22 37 20 13 64 35	130 Francony 1881 7 54 43 28 1 30 47 50 42 27 6 53 48 51 2 31 55 8 25 44 29 4 49 46 26 41 56 5 52 45 32 3 39 24 9 60 33 62 15 18 10 57 40 21 16 19 34 63 23 38 59 12 61 36 17 14 58 11 22 37 20 13 64 35	131 Francony 1882 7 54 43 28 1 30 47 50 42 27 6 53 48 51 2 31 55 8 25 44 29 4 49 46 26 41 56 5 52 45 32 3 9 24 39 60 33 62 15 18 40 57 10 21 16 19 34 63 23 38 59 12 61 36 17 14 58 11 22 37 20 13 64 35	132 Murray 1936 7 30 43 52 1 54 47 26 42 51 6 29 48 27 2 55 31 8 49 44 53 4 25 46 50 41 32 5 28 45 56 3 9 60 39 22 33 58 15 24 40 21 10 59 16 23 34 57 61 38 19 12 63 36 17 14 20 11 62 37 18 13 64 35	133 Murray 1936 7 30 43 52 1 54 47 26 42 51 6 29 48 27 2 55 31 8 49 44 53 4 25 46 50 41 32 5 28 45 56 3 39 60 9 22 33 58 15 24 10 21 40 59 16 23 34 57 61 38 19 12 63 36 17 14 20 11 62 37 18 13 64 35	134 Exner 1876 31 34 47 20 1 22 51 54 46 19 32 35 50 53 2 23 33 30 17 48 21 4 55 52 18 45 36 29 56 49 24 3 43 64 9 16 37 28 5 58 12 15 44 61 8 57 38 25 63 42 13 10 27 40 59 6 14 11 62 41 60 7 26 39
135 Exner 1876 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 4 45 26 50 13 36 29 48 25 56 3 11 64 17 40 21 58 5 44 18 37 12 61 8 43 24 57 63 10 39 20 41 22 59 6 38 19 62 9 60 7 42 23	136 Ligondès 1883 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 4 45 26 50 13 36 29 48 25 56 3 11 64 17 40 21 58 5 44 18 37 12 61 8 43 24 57 63 10 39 20 41 22 59 6 38 19 62 9 60 7 42 23	137 Ligondès 1883 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 26 45 4 50 13 36 29 48 3 56 25 11 64 17 40 21 58 5 44 18 37 12 61 8 43 24 57 63 10 39 20 41 22 59 6 38 19 62 9 60 7 42 23	138 Ligondès 1883 31 34 15 52 1 54 27 46 14 51 30 33 28 47 2 55 35 32 49 16 53 26 45 4 50 13 36 29 48 3 56 25 11 64 17 40 21 58 5 44 18 37 12 61 8 43 24 57 63 10 39 20 41 22 59 6 38 19 62 9 60 7 42 23	139 Ligondès 1883 47 22 39 28 1 50 43 30 38 25 48 23 44 29 2 51 21 46 27 40 49 4 31 42 26 37 24 45 32 41 52 3 15 20 33 60 5 64 9 54 36 59 14 17 12 63 6 53 19 16 57 34 61 8 55 10 58 35 18 13 56 11 62 7	140 Ligondès 1883 55 30 47 4 57 6 51 10 46 1 56 31 52 9 58 7 29 54 3 48 5 60 11 50 2 45 32 53 12 49 8 59 39 28 41 20 33 24 61 14 44 19 38 25 64 13 34 23 27 40 17 42 21 36 15 62 18 43 26 37 16 63 22 35